University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2000

Problem Set 2

Fourier Transform, Hilbert Transform, Bandpass Systems

Issued: Thursday, Sept. 7th. **Due:** Thursday, Sept. 14th (beginning of lecture).

Reading from Lathi: Chapter 2 and Chapter 3 (Sections 3.1–3.5).
Reading from Haykin: Chapter 2 (Sections 2.9–2.14).

Problem 2.1

(a) Problem 3.3-4 from Lathi, pp. 145–146.

(b) Problem 3.3-6(a) from Lathi, p. 146.

(c) **(optional)** Problem 3.3-6(b,c) from Lathi, p. 146.

Problem 2.2 (Optional)

Problem 3.3-7 from Lathi, p. 146.

Problem 2.3

Problem 3.7-2 from Lathi, p. 149.

Problem 2.4

(a) Show that the complex envelope of the sum of two narrow-band signals (with the same carrier frequency) is equal to the sum of their individual complex envelopes.

(b) Consider a signal of the form

$$s(t) = c(t)m(t) \,,$$

where $m(t)$ is a low-pass signal whose Fourier transform $M(f)$ is zero for $|f| > W$, and $c(t)$ is a high-pass signal whose Fourier transform $C(f)$ is zero for $|f| < W$. Show that the Hilbert transform of $s(t)$ is given by

$$\hat{s}(t) = \hat{c}(t)m(t) \,,$$

where $\hat{c}(t)$ is the Hilbert transform of $c(t)$.

Problem 2.5

Let $m(t) = \text{sinc}(t)$ and let $\widehat{m}(t)$ denote its Hilbert transform. Define

$$x(t) = m(t) \cos \omega_0 t - \widehat{m}(t) \sin \omega_0 t$$

to be a bandpass signal ($\omega_0 >> \pi$).

(a) Find the pre-envelope $x_+(t)$ and the complex envelope $\widetilde{x}(t)$ of signal $x(t)$.

(b) Determine the Fourier transform and bandwidth of $x(t)$.

Problem 2.6

Problem 3.3-10 from Lathi, p. 147.