University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2000

Problem Set 3

Narrowband Signals Through LTI Systems, Amplitude Modulation

Issued: Thursday, Sept. 14th. **Due:** Thursday, Sept. 21st (beginning of lecture).

Reading from Lathi: Chapter 3, Sections 3.5–3.6 and Chapter 4, Sections 4.1–4.4.
Reading from Haykin: Chapter 2, Sections 2.13–2.14 and Chapter 3, Sections 3.1–3.4 and 3.8–3.9.

Problem 3.1

The rectangular pulse

$$x(t) = \begin{cases} A\cos(\omega_0 t) , & 0 \le t \le T \\ 0 , & \text{otherwise} \end{cases}$$

goes through an LTI system with impulse response

$$h(t) = x(T - t) \ .$$

Find the output $y(t)$ of the filter under the assumption that the frequency ω_0 is a large integer multiple of $\frac{2\pi}{T}$.

Problem 3.2 (Optional)

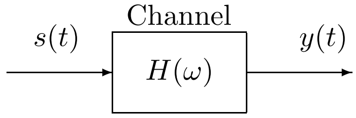
Problem 3.5-4 from Lathi, p. 148 (assume that distortionless transmission requires an amplitude variation of less than 4% and a time delay variation of less than 2%).

Problem 3.3

Suppose the modulated signal $s(t) = m(t)\cos(\omega_c t)$ is applied as an input to an LTI communication channel with frequency response $H(\omega)$, where the modulating signal $m(t)$ is given by

$$m(t) = \mathrm{sinc}(t/T) \ .$$

Assume $(1/T) = 75$ kHz and $\omega_c = 2\pi \times 1300$ kHz.



(a) Make a neat and fully labeled sketch of $S(\omega)$.

(b) Find a time-domain expression for the output $y(t)$ of the channel if the channel frequency response is

$$H(\omega) = e^{-j\omega(4\times 10^{-6})} .$$

(c) Find an *approximate* (but reasonably accurate) time-domain expression for the output $y(t)$ of the channel if the channel characteristics are actually as shown in Figure 3.3-1 (on the last page of the problem set) rather than as specified in (b). Also state what features of the signal and/or channel make your approximation reasonable.

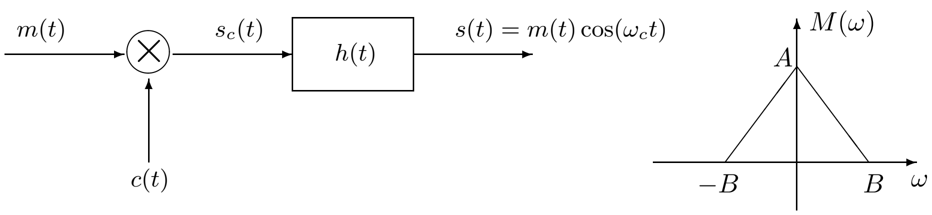
Problem 3.4 (Optional)

Problem 4.2-1 from Lathi, p. 202.

Problem 3.5

The spectrum of the input signal $m(t)$ and a DSB-SC modulator are shown below. The carrier $c(t)$ available at the multiplier is *distorted* and is given by

$$c(t) = a_1 \cos(\omega_c t) + a_2 \cos^2(\omega_c t) .$$



(a) Determine the spectrum of the signals $s_c(t)$ and $s(t)$.

(b) What constraints should the filter $h(t)$ satisfy so that the transmitted signal $s(t)$ is the desirable one?

(c) What minimum value of ω_c is required for this system to work?

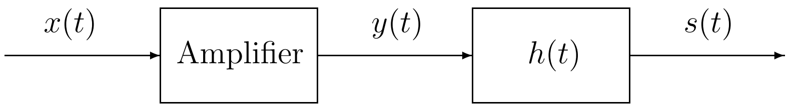
Problem 3.6

Problem 4.2-8 from Lathi, p. 203.

Problem 3.7

A DSB-SC signal $x(t) = m(t)\cos(\omega_c t)$ is amplified before transmitted over a channel. Unfortunately, the amplifier is nonlinear with output $y(t)$ related to its input $x(t)$ via the relation

$$y(t) = 100x(t) + x^2(t) \ .$$



(a) Assuming the spectrum of $m(t)$ is limited to $\pm B$ rad/s, find and sketch the spectra of signals $y(t)$ and $s(t)$. (Filter $h(t)$ is the standard bandpass filter used in DSB-SC modulation.)

(b) If we use coherent detection (i.e., assuming we know ω_c exactly), is it possible to recover signal $m(t)$ at the receiver without distortion? If so, what are the restrictions on the value of ω_c ?

Problem 3.8 (Optional)

A square-law detector is one that uses a nonlinear device to demodulate an amplitude modulated waveform. The output $y(t)$ of this nonlinear device is related to its input $x(t)$ via

$$y(t) = x(t) + \alpha x^2(t) \ ,$$

where α is a constant.

If the input to this nonlinear device is an amplitude modulated signal

$$s(t) = A_c[1 + km(t)]\cos(\omega_c t) \ ,$$

find (i) the output $y(t)$ and (ii) the conditions under which $m(t)$ can be recovered exactly from $y(t)$.

Figure 3.3-1: Filter characteristics for Problem 3.3.