University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2001

# Problem Set 1

## Introduction, Linear Time-Invariant Systems, Fourier Transform

**Issued:** Tuesday, August 28th. **Due:** Beginning of lecture on Thursday, September 6th.

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**Reading from Haykin (4th Edition):** *Background and Preview* and Appendix II. Most of the material in Appendix II should be familiar to you from previous courses.

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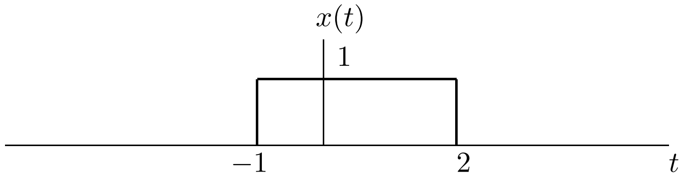
### Problem 1.1

(a) Consider an LTI system with input $x(t)$ and output $y(t)$ related through the equation

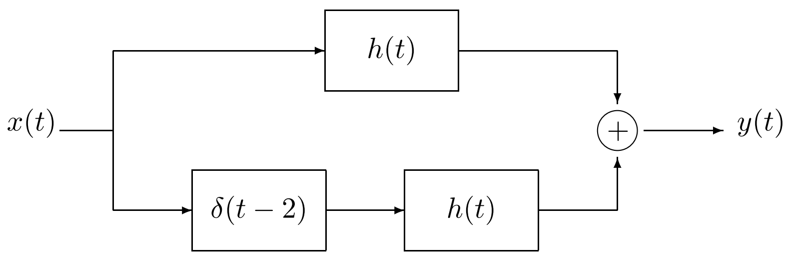
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 5) d\tau.$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response $y(t)$ of the system in part (a) when the input $x(t)$ is as shown below.



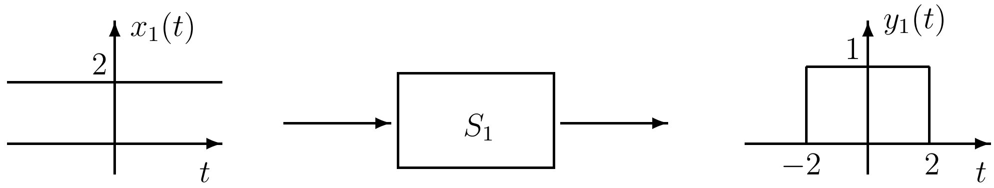

(c) Consider the following interconnection of LTI systems:




Here, $h(t)$ is as in part (a). Determine the output $y(t)$ when the input $x(t)$ is given as in part (b). (Hint: You do *not* need to evaluate a convolution integral.)

### Problem 1.2

(a) The following system $S_1$ (*not necessarily* LTI) is known to have the input-output pair shown below:




Is system $S_1$ time-invariant? Explain.

(b) A *nonlinear* system $S_2$ is known to be time-invariant. Is the output $y_2(t)$ of system $S_2$ guaranteed to be periodic in $t$ when input $x_2(t) = \cos(2\pi f_0 t)$ is applied? Explain.

### Problem 1.3

Find the energy and half-power (3dB) bandwidth of the signal $x(t) = e^{-5t}u(t)$.

### Problem 1.4

Show that the Fourier transform of signal $x(t)$ can be written as

$$X(f) = \int_{-\infty}^{\infty} x(t)\cos(2\pi f t)dt - j\int_{-\infty}^{\infty} x(t)\sin(2\pi f t)dt \ .$$

Using the above, show that if $x(t)$ is an even function of $t$, then

$$X(f) = 2\int_{0}^{\infty} x(t)\cos(2\pi f t)dt \ .$$

Also, show that a signal $x(t)$ that is a *real and even* function of $t$ has Fourier transform $X(f)$ that is a *real and even* function of $f$.

### Problem 1.5

Consider the signal $x(t) = A \text{ rect}\left(\frac{t}{T} - \frac{1}{2}\right)$. Find the Fourier transforms of the even part $x_e(t)$ and the odd part $x_o(t)$ of $x(t)$, defined as

$$\begin{aligned} x_e(t) &= \frac{1}{2}\left[x(t) + x(-t)\right] \ , \\ x_o(t) &= \frac{1}{2}\left[x(t) - x(-t)\right] \ . \end{aligned}$$

**Problem 1.6**

A signal $x(t)$ is applied to a square-law device whose output $y(t)$ is defined by

$$y(t) = x^2(t) \ .$$

If the spectrum of $x(t)$ is limited to the frequency interval $-W \leq f \leq W$, show that the spectrum of $y(t)$ is limited to $-2W \leq f \leq 2W$.

**Problem 1.7 (Optional)**

(a) Find the Fourier transform of the *Gaussian* signal

$$x(t) = Ae^{-t^2/2T^2} \ , \quad -\infty < t < +\infty \ .$$

(b) If this signal goes through an LTI system with frequency response

$$H(f) = e^{-\frac{2\pi f}{2B^2}} \ , \quad -\infty < f < +\infty \ ,$$

what is the output $y(t)$? Simplify the expression for $y(t)$ for $T >> 1/B$.

**Problem 1.8**

A linear time-invariant system $S$ is known to be stable. Show that, if the input $x(t)$ of $S$ has finite energy, then the output $y(t)$ also has finite energy. In other words, show that if

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty \ ,$$

then

$$\int_{-\infty}^{+\infty} |y(t)|^2 dt < \infty \ .$$