University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2001

Problem Set 5

PM and FM Modulation, Phase-Locked Loops, Probability Review

Issued: Tuesday, October 9th. **Due:** Thursday, October 18th (beginning of lecture).

Reading from Haykin: Chapter 2, Sections 2.6–2.9, and Appendix 1.

Problem 5.1

Consider the FM signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right] ,$$

where $m(t) = A_m \sin(2\pi f_m t)$ and

$$A_c = 200V, \quad f_c = 80 \text{ MHz}, \quad k_f = 10 \text{ KHz/Volt}, \quad A_m = 1V, \quad f_m = 8 \text{ KHz} .$$

(a) Find the frequency deviation Δf.

(b) Find the modulation index β.

(c) The narrow-band FM signal

$$s_e(t) = A_c \cos(2\pi f_c t) - A_c \left[2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right] \sin(2\pi f_c t)$$

is used as an approximation to $s(t)$. Sketch $|S_e(f)|$ and find its bandwidth.

(d) Find the power of $s_e(t)$ and compare it to the power of $s(t)$. How accurately does the narrow-band FM signal $s_e(t)$ approximate $s(t)$ in this case?

Problem 5.2 (Optional)

Recall that the complex envelope $\tilde{s}(t)$ of a real signal $s(t)$ can be written in the form

$$\tilde{s}(t) = s_I(t) + j s_Q(t) ,$$

where $s_I(t)$ and $s_Q(t)$ are the in-phase and quadrature-phase components of $s(t)$.

Find expressions for the in-phase and quadrature-phase components of the following signals:

(a) PM signal $s(t) = A\cos[2\pi f_c t + k_p m(t)]$.

(b) FM signal $s(t) = A\cos[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau]$.

Problem 5.3

Given $m(t) = \sin(2000\pi t)$, $k_f = 10^5$ and $k_p = 10$, answer the following questions.

(a) Estimate the bandwidth of $s_{FM}(t)$ and $s_{PM}(t)$.

(b) Repeat part (a) if the amplitude of $m(t)$ is doubled.

(c) Repeat part (a) if the frequency of $m(t)$ is doubled.

Problem 5.4

Problem 2.33 from Haykin, p. 175.

Problem 5.5

Problem 2.39 from Haykin, p. 177.

Problem 5.6

Problem 2.40 from Haykin, p. 177.

Problem 5.7

A binary source generates digits "1" and "0" with probabilities .7 and .3 respectively. What is the probability that a consecutive stream of ten "1s" is observed? What is the probability that the string "1010101010" is observed?

Problem 5.8

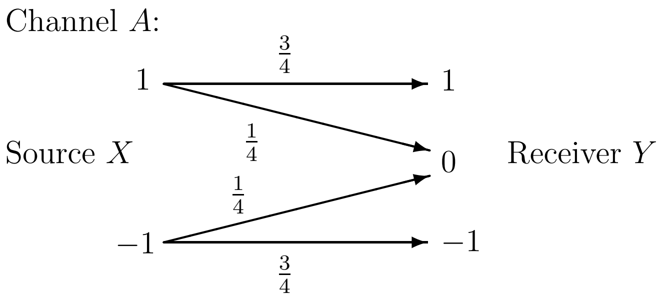
A communication system transmits symbols labeled "1," "2," and "3." The joint probability that symbol t is sent *and* symbol r is received is given in the following table for each pair (t, r) of sent and received symbols. For example, the probability that a "1" is sent and a "3" is received (due to noise in the channel/system) is given by 0.10.

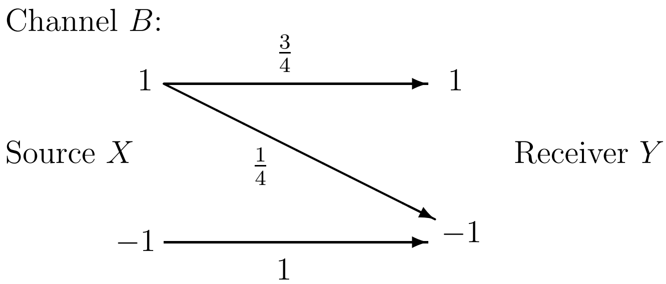
	r		
t	1	2	3
1	0.13	0.05	0.10
2	0.08	0.16	0.05
3	0.10	0.13	0.20

Calculate the probability that symbol k was sent, given that symbol k was received (for $k = 1, 2, 3$). Also calculate the probability of error in this system (an error is defined as the reception of a signal other than the symbol that was transmitted).

Problem 5.9

A particular "antipodal" *pulse amplitude modulation* (PAM) communication system has a transmitter ("source") that transmits the symbol $X = -1$ and the symbol $X = +1$ with equal probability. This source can use one of two channels, Channel A and Channel B. The figures below show the characteristics of each of these channels (the number next to an arrow denotes the probability of receiving the symbol at the right of the arrow, given transmission of the symbol at the left of the arrow). Channel A occasionally looses the symbol completely, while for Channel B the symbol $X = 1$ is at times misinterpreted as a -1 at the receiver. The receiver does not know explicitly which channel was used.





(a) If the source uses Channel A, what is the probability that $Y = -1$ is received? What are the probabilities of receiving $Y = 0$ and $Y = 1$? Repeat your calculations for the case when Channel B is used.

(b) Assume now that the source uses Channel A with probability α and Channel B with probability $1 - \alpha$. Suppose that the received symbol is $Y = -1$. What is the probability that Channel A was used, given that a $Y = -1$ was received?

(c) Suppose that you use the *maximum a posteriori probability* (MAP) rule to decide whether Channel A or Channel B was used. In other words, if $Y = -1$, you decide in favor of Channel A if

$$\Pr(A \text{ used} \mid Y = -1) > \Pr(B \text{ used} \mid Y = -1) \ .$$

Similarly, if $Y = 0$, you decide in favor of Channel A if

$$\Pr(A \text{ used} \mid Y = 0) > \Pr(B \text{ used} \mid Y = 0) \ ,$$

and, if $Y = 1$, you decided in favor of Channel A if

$$\Pr(A \text{ used} \mid Y = 1) > \Pr(B \text{ used} \mid Y = 1) \ .$$

What does the MAP rule reduce to in this case? For what range of α will you decide that Channel A was used, regardless of the value of Y that is received?