University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 1999

Mid-Semester Exam I

Thursday, September 30, 1:30–2:50pm, 151 Everitt Laboratory

READ THESE COMMENTS BEFORE STARTING THE EXAM!!

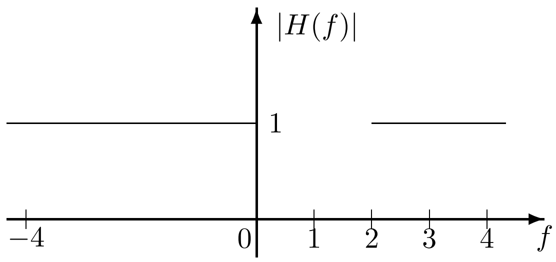
- This is a **closed-book** exam, but **one** sheet of notes (both sides) is allowed. Calculators should not be necessary, but feel free to use one.
- **Write your name on the answer booklet.**
- There are **five** problems. The weighting is indicated within each problem.
- A correct answer does not guarantee credit; an incorrect answer does not guarantee loss of credit. **Provide clear explanations, show all relevant work and justify your answers!** If we cannot make sense of your writing or reasoning, you may lose points.
- Read each problem carefully and *think* before performing detailed calculations.
- Only the supplied answer booklet is to be handed in. **No additional pages will be considered in the grading.** You may want to work things through in the blank areas of the exam and then neatly transfer to the answer sheet the work you would like us to look at.

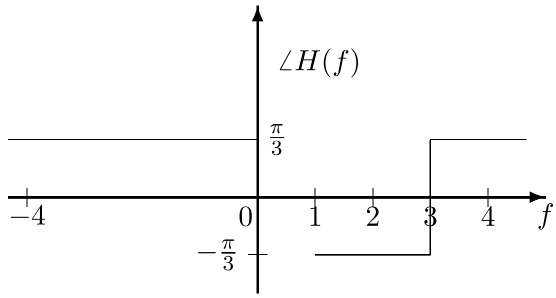
Useful Formulas:

- $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$
- $\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
- $\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$
- $\cos a \cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$
- $\sin a \cos b = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$
- $\cos^2(2\theta) = \cos^2\theta - \sin^2\theta$
- $\sin^2\theta + \cos^2\theta = 1$
- $e^{j\theta} = \cos\theta + j\sin\theta$
- $\mathcal{FT}\{e^{-\alpha t}u(t)\} = \frac{1}{\alpha + j2\pi f}, \quad \alpha > 0$
- $\mathcal{FT}\{\mathrm{sinc}(2Wt)\} = \frac{1}{2W}\mathrm{rect}(\frac{f}{2W})$
- $\mathcal{HT}\{\cos(2\pi f_0 t)\} = \sin(2\pi f_0 t)$

Problem 1 (10/80, equally weighted parts)

An LTI system with impulse response $h(t)$ has the following frequency response $H(f)$:





(a) Determine whether the following statement is TRUE or FALSE:

The impulse response $h(t)$ of the system is purely real.

Justify your answer.

(b) If the input to the system is $\cos(2\pi f_0 t)$ with $0 < f_0 < 4$, for what values of f_0 will the output be real-valued?

Hint: For the output to be $|H(f_0)| \cos(2\pi f_0 t + \angle H(f_0))$, the impulse response of the system has to be real-valued.

Problem 2 (10/80, equally weighted parts)

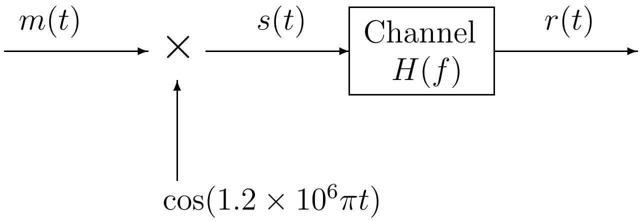
Consider the signal $g(t) = e^{-3t}u(t)$.

(a) Find the energy of $g(t)$?

(b) Find the half-power (3dB) bandwidth of $g(t)$.

Problem 3 (25/80, equally weighted parts)

A message signal $m(t)$ is transmitted via a communication channel using an amplitude modulation scheme as shown in the following diagram:



We model the communication channel as an LTI system whose frequency response $H(f)$ has the characteristics shown in Figure 1. The Fourier transform of the signal $m(t)$ is given by

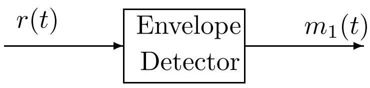
$$M(f) = \begin{cases} 1\ , & |f| \leq 500\text{Hz} \\ 0\ , & |f| > 500\text{Hz}\ . \end{cases}$$

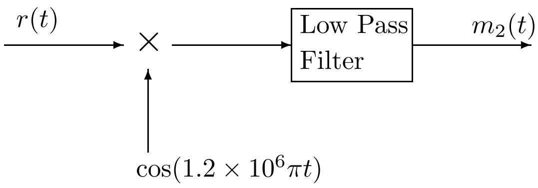
(a) Find and sketch $m(t)$.

(b) Provide an accurate sketch of $S(f)$.

(c) Find a reasonable approximation to the received signal $r(t)$ (the output of the communication channel).

The signal $r(t)$ is demodulated in two different ways. In one case, we use a coherent detector and in the other case we use an envelope detector:





(d) Find and sketch $m_1(t)$.

(e) Find and sketch $m_2(t)$.

Hint: If you have not been able to find $r(t)$ in part (c), you can still receive partial credit for parts (d) and (e). Assume that $r(t)$ is given by $r(t) = m(t)\cos(2\pi f_c t + \phi_0)$ and solve for $m_1(t)$ and $m_2(t)$.

Figure 1: Communication channel characteristics for Problem 3.

Problem 4 (15/80, equally weighted parts)

In this problem, we consider the upper sideband SSB signal $s_u(t)$, generated by modulating a carrier at frequency $f_c = 1\text{MHz}$ with the message signal $m(t) = \cos(2000\pi t)$.

(a) Sketch the spectrum of the DSB-SC signal $s(t) = m(t)\cos(2\pi f_c t)$.

(b) Sketch the spectrum $S_u(f)$ of the corresponding upper sideband SSB signal. By inverting this spectrum, find $s_u(t)$.

(c) Verify that the time domain expression for $s_u(t)$ in part (b) is equal to

$$2s_u(t) = m(t)\cos(2\pi f_c t) - \widehat{m}(t)\sin(2\pi f_c t) \ ,$$

where $\widehat{m}(t)$ is the Hilbert transform of $m(t)$.

Problem 5 (20/80, equally weighted parts)

Consider the FM signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau\right] .$$

where $m(t)$ is the tone signal $m(t) = A_m \sin(2\pi f_m t)$. The following parameters are given:

$$A_c = 200V, \quad f_c = 80\text{MHz}, \quad k_f = 10\text{KHz/Volt}, \quad A_m = 1V, \quad f_m = 8\text{KHz} .$$

(a) Find the frequency deviation Δf.

(b) Find the modulation index β.

(c) The narrowband FM signal approximation uses

$$\widetilde{s}(t) = A_c \cos(2\pi f_c t) - A_c \left[2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau\right] \sin(2\pi f_c t)$$

as an approximation to $s(t)$. Sketch $|\widetilde{S}(f)|$ and find its bandwidth.

(d) Find the power of $\widetilde{s}(t)$ and compare it to the power of $s(t)$.