University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2001

Mid-Semester Exam I

Thursday, September 27, 1:30–2:50pm, 161 Everitt Laboratory

READ THESE COMMENTS BEFORE STARTING THE EXAM!!

- This is a **closed-book** exam, but **one** sheet of notes (both sides) is allowed. Calculators should not be necessary, but feel free to use one.
- **Write your name on the answer booklet.**
- There are **four unequally weighted** problems for a total of **80 points**. Problems are *not* necessarily in order of difficulty.
- A correct answer does not guarantee credit; an incorrect answer does not guarantee loss of credit. **Provide clear explanations, show all relevant work and justify your answers!** If we cannot make sense of your writing or reasoning, you may lose points.
- Read each problem carefully and *think* before performing detailed calculations.
- Only the supplied answer booklet is to be handed in. **No additional pages will be considered in the grading.** You may want to work things through in the blank areas of the exam and then neatly transfer to the answer sheet the work you would like us to look at.

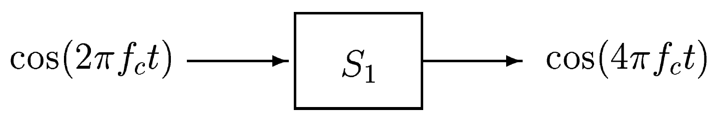
Formulas:

- $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$
- $\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
- $\sin a \sin b = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$
- $\cos a \cos b = \frac{1}{2}\left[\cos(a-b) + \cos(a+b)\right]$
- $\sin a \cos b = \frac{1}{2}\left[\sin(a-b) + \sin(a+b)\right]$
- $\cos^2(2\theta) = \cos^2\theta - \sin^2\theta$
- $\sin^2\theta + \cos^2\theta = 1$
- $e^{j\theta} = \cos\theta + j\sin\theta$
- $\mathcal{FT}\{e^{-\alpha t}u(t)\} = \frac{1}{\alpha + j2\pi f}, \quad \alpha > 0$
- $\mathcal{FT}\{\text{sinc}(2Wt)\} = \frac{1}{2W}\text{rect}(\frac{f}{2W})$
- $\mathcal{HT}\{\cos(2\pi f_c t)\} = \sin(2\pi f_c t)$

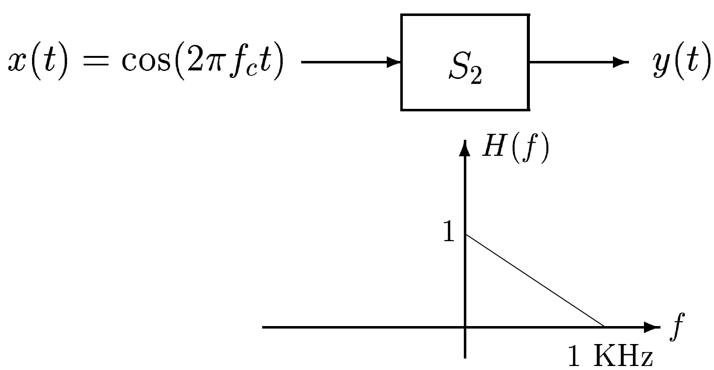
Problem 1 (20/80, equally weighted parts)

This problem has **two independent** parts.

Part A: Consider the system S_1 shown below with the indicated input/output pair and $f_c = 1$ Hz. Determine whether the system (i) could not, (ii) could or (iii) must be linear time-invariant (LTI). Choose the strongest statement that applies and *justify* your answer. If you decide that the system could or must be LTI, determine a possible frequency response for it.



Part B: The input signal $x(t) = \cos(2\pi f_c t)$ with $f_c = 500$ Hz is applied to the linear time-invariant (LTI) system S_2 with frequency response $H(f)$ as shown below. Determine $y(t)$.



Problem 2 (15/80, equally weighted parts)

This problem has **three independent** parts.

Part A: Find the energy of the signal $x(t)$ given by

$$x(t) = e^{-\alpha t}u(t) + e^{\alpha t}u(-t)$$

for $\alpha > 0$.

Part B: Find the half-power bandwidth of the signal $x(t)$ given by

$$x(t) = e^{-\alpha t}u(t) + e^{\alpha t}u(-t)$$

for $\alpha > 0$.

Part C: The message signal $m(t) = A_m \sin(2\pi f_m t)$ is modulated using DSB-SC modulation so that the resulting signal is

$$s(t) = m(t)\cos(2\pi f_c t)$$

with $f_c >> f_m$.

What is the output $y(t)$ if the signal $s(t)$ is demodulated using an envelop detector?

Problem 3 (15/80)

The input signal $x(t)$ is of the form

$$x(t) = m(t)\cos(2\pi f_c t)$$

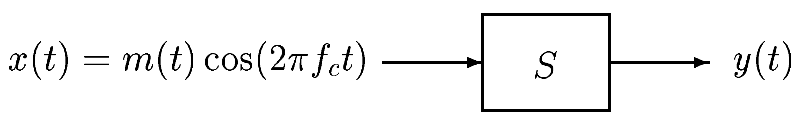
and is applied to the linear time-invariant (LTI) system S with impulse response

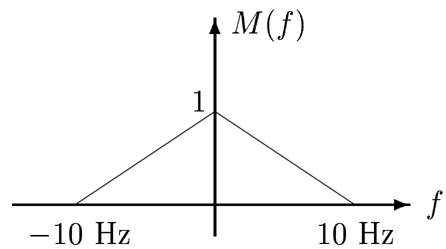
$$h(t) = e^{-\alpha t}u(t) \quad , \alpha = 10^3 \ ,$$

as shown below.

Given that $m(t)$ is a real signal with Fourier transform $M(f)$ as shown below and given that $f_c = \frac{1}{2\pi}$ KHz, find an *approximate* expression for the output $y(t)$ (in terms of $m(t)$).

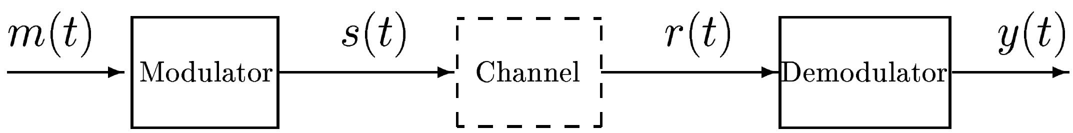
Hint: Note that $\frac{d}{dx}\left\{\tan^{-1}(x)\right\} = \frac{1}{1+x^2}$.





Problem 4 (30/80, equally weighted parts)

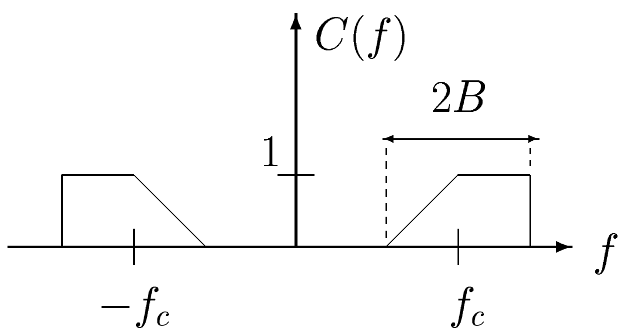
This problem has **three relatively independent** parts.



Consider a voice message $m(t)$ that is low-pass with bandwidth $B = 5$ KHz. This signal is modulated using DSB-SC so that the transmitted signal $s(t)$ is given as

$$s(t) = m(t)\cos(2\pi f_c t)$$

with $f_c = 20$ KHz. The signal $s(t)$ is transmitted through a communication channel with frequency response $C(f)$ as shown below.



(a) Using only oscillators, ideal filters and amplifiers, devise a demodulation scheme to obtain $m(t)$ from the signal $r(t)$ arriving at the receiver.

(b) An ECE359 student claims that the demodulation at the receiver can be simplified if we are willing to pre-filter the signal $s(t)$ before transmission (i.e., if we are willing to insert a pre-filter $H(f)$ before transmitting through the channel). Do you agree or disagree with this statement? If you agree, provide an appropriate pre-filter $H(f)$ and the corresponding (simplified) demodulation scheme.

(c) Another ECE359 student claims that the demodulation at the receiver can be simplified if we are willing to use a different modulating frequency f_c. Do you agree or disagree with this statement? If you agree, provide an appropriate value for f_c and the corresponding (simplified) demodulation scheme.