University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2002

Problem Set 3

Amplitude Modulation Schemes

Issued: Thursday, Sept. 19th. **Due:** Thursday, Sept. 26th (beginning of lecture).

Reading from Proakis (2nd Edition): Chapter 3, Sections 3.1 and 3.2.

Announcement: The first Mid-Semester Exam will be held on Thursday, October 3rd, from 1:30pm to 2:50pm in 165 Everitt. The exam will cover all material from the beginning of the term *up to and including* the lecture on Thursday, September 26th. The corresponding material will include Problem Sets 1 through 4 and the following reading from Proakis (2nd Edition): Chapters 1, 2 and 3 (up to and including Section 3.3.2).
During the exam, you can bring an 8.5×11-inch double-sided sheet of *handwritten* notes. Calculators are allowed but will not be necessary.
A copy of an old exam will be available from `http://www.ece.uiuc.edu/ece359` . This sample exam does not necessarily resemble this year's exam (also notice that the material covered in this old exam is slightly different from the material covered in this year's exam).

Problem 3.1

Problems 3.1 and 3.2 from Proakis (2nd Edition), p. 131.

Problem 3.2

Problem 3.7 from Proakis (2nd Edition), p. 132.

Problem 3.3 (Optional)

A square-law detector is one that uses a nonlinear device to demodulate an amplitude modulated waveform. The output $y(t)$ of this nonlinear device is related to its input $x(t)$ via

$$y(t) = x(t) + \alpha x^2(t) ,$$

where α is a constant.
If the input to this nonlinear device is an amplitude modulated signal

$$s(t) = A_c[1 + km(t)] \cos(2\pi f_c t) ,$$

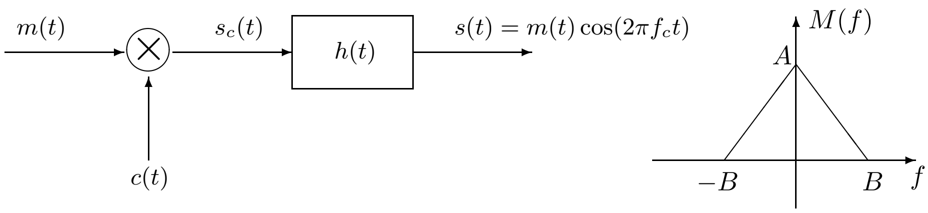
find (i) the output $y(t)$ and (ii) the conditions under which $m(t)$ can be recovered exactly from $y(t)$.

Problem 3.4

The spectrum of the input signal $m(t)$ and a DSB-SC modulator are shown below. The carrier $c(t)$ available at the multiplier is *distorted* and is given by

$$c(t) = a_1 \cos(2\pi f_c t) + a_2 \cos^2(2\pi f_c t) \ .$$

The filter $h(t)$ needs to be chosen so that $s(t) = m(t)\cos(2\pi f_c t)$.



(a) Determine the spectrum of the signals $s_c(t)$ and $s(t)$.

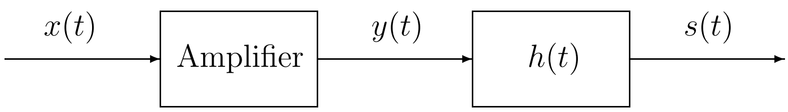
(b) What constraints should the filter $h(t)$ satisfy so that the transmitted signal $s(t)$ is the desirable one?

(c) What minimum value of f_c is required for this system to work?

Problem 3.5

A DSB-SC signal $x(t) = m(t)\cos(2\pi f_c t)$ is amplified before transmitted over a channel. Unfortunately, the amplifier is nonlinear with output $y(t)$ related to its input $x(t)$ via the relation

$$y(t) = 100x(t) + x^2(t) \ .$$



(a) Assuming the spectrum of $m(t)$ is limited to $\pm B$ Hz, find and sketch the spectra of signals $y(t)$ and $s(t)$. (Filter $h(t)$ is the standard bandpass filter used in DSB-SC modulation.)

(b) If we use coherent detection (i.e., assuming we know f_c exactly), is it possible to recover signal $m(t)$ at the receiver without distortion? If so, what are the restrictions on the value of f_c ?

Problem 3.6 (Optional)

The complex envelope $\widetilde{x}(t)$ of a real signal $x(t)$ can be written in the form

$$\widetilde{x}(t) = x_I(t) + jx_Q(t) ,$$

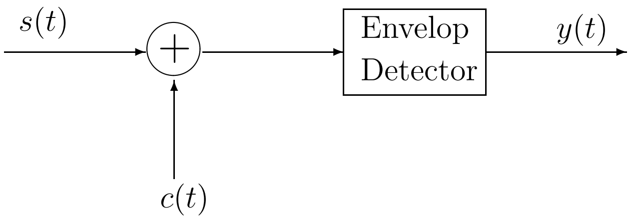
where $x_I(t)$ and $x_Q(t)$ are the in-phase and quadrature-phase components of $x(t)$.
Find the expressions for $x_I(t)$ and $x_Q(t)$ of the following signals:

(a) DSB-SC signal $x(t) = m(t)\cos(2\pi f_c t + \phi)$.

(b) DSB-TC signal $x(t) = [A + m(t)]\cos(2\pi f_c t + \phi)$.

Problem 3.7

Show that a DSB-SC signal can be demodulated by an envelope detector if a sufficient amount of carrier is inserted at the receiver as shown below.



In particular, show that if the carrier $c(t) = A\cos[2\pi(f_c + \Delta f)t + \phi]$ is inserted in the received DSB-SC signal $s(t) = m(t)\cos 2\pi f_c t$ and the resulting signal is envelope-detected, then the output (after DC blocking) will be $y(t) = m(t)\cos(2\pi\Delta f t + \phi)$.