ECE 459: Solution for Sample Final Exam

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Problem 1

(a) Averaged power=3. $F_Y(y) = \frac{1}{\sqrt{2\pi}\sqrt{3}} e^{-\frac{y^2}{6}}$
(b) $\hat{Y}_{LMMSE}(x) = \frac{2}{3}x$

Problem 2

(a) Signal power=500 and noise power=10. So $SNR = 50$.
(b) Signal power=250 and noise power=2.5. So $SNR = 100$.

Problem 3

Part a.

(a) $\psi(t) = k_f \int_{-\infty}^{t} m(\tau)d\tau$
(b) The power spectral density is $\frac{N}{A^2} rect(\frac{\omega}{4\pi B})$ and the averaged power is $\frac{2NB}{A^2}$
(c) $D(\omega) = j\omega$ and $SNR = \frac{k_f^2 P(3A^2)}{8\pi^2 NB^3}$

Part b.

(d) $D(\omega) = \omega^2$ and $S_{w'w'}(\omega) = \omega^4(\frac{N}{A^2})$ from $-2\pi B$ to $2\pi B$.
(e) $SNR = \frac{k_f^2 P(5A^2)}{32\pi^4 NB^5}$ and the range for B is $2\pi B \leq \sqrt{5/3}$

Problem 4

Part a.

(a) $f_{Y|H_0}(y|H_0) \sim N(0, N/2)$ and $f_{Y|H_1}(y|H_1) \sim N(1/2, N/2)$
(b) $\gamma = 1/4$ and $P_e = Q(\frac{1/4}{\sqrt{N/2}})$
(c) Yes. Choose $h(t) = x(1-t)$ from 0 to 1 where $x(t) = p(t) * c(t)$ (because the filter has to be causal). The resulting γ is 1/6 and $P_e = Q(\frac{1/6}{\sqrt{N/6}})$.

Part b.

(d)(e) Choose $T = 2$ and $h(t) = x(2-t)$. The resulting γ is 1/3 and $P_e = Q(\frac{1/3}{\sqrt{N/3}})$.

Problem 5

A. TRUE. Passing $x(t)$ through $g(t)$ to get $z(t)$ is equivalent to passing $x(t)$ through $h(t)$ to get $y(t)$ and then passing $y(t)$ through another $h(t)$ to get $z(t)$. Since $h(t)$ is BIBO stable, if $x(t)$ is finite in energy, $y(t)$ is also finite in energy and so is $z(t)$. This implies that $g(t)$ is also BIBO.

B. FALSE. $\hat{Y}_{LMMSE}(x)$ should be 0.

C. FALSE. The counter example is to choose the joint density $f_{X,Y}(x, y) = 1$ for the region $[-1/2, 1/2] \times [-1/2, 1/2]$. Then we can find that $E[Y|X]$ is the same as $\hat{Y}_{LMMSE}(x)$ and both equal to 0.

D. FALSE. $\sigma_Y^2 = \sigma_{X_1}^2 + 2\sigma_{X_1X_2} + \sigma_{X_2}^2$ not necessarily equals to 2.

E. TRUE. Since $S_{XX}(\omega)$ is the sinc function and has negative parts, which is not permissible for PSD.

F. TRUE. $R_{XX}(2) = 0 \rightarrow$ uncorrelated $\rightarrow$ independent. (b/c they are Gaussian)