University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 459: Communications I

Fall 2004

Problem Set 3

Amplitude Modulation, Angle Modulation

Issued: Tuesday, Sept. 21st. **Due:** Thursday, Sept. 30th (beginning of lecture).

Reading from Lathi: Chapter 4, Sections 4.1–4.6.

Reading from Haykin: Chapter 2, Sections 2.1–2.6.

Announcement: The first Mid-Semester Exam will be held on Tuesday, October 5th, from 5-7pm in 163 Everitt. The exam will cover all material from the beginning of the term *up to and including* the lecture on Tuesday, September 28th. The corresponding material includes Problem Sets 1 through 3 and:
(i) **Lathi**: Chapters 1, 2, 3, and 4 (excluding Sections 4.7–4.9), OR
(ii) **Haykin**: Appendix 2 and Chapters 0 and 2 (excluding Sections 2.7–2.15).
During the exam, you can bring an 8.5 × 11-inch double-sided sheet of *handwritten* notes. Calculators are allowed but will not be necessary.
A copy of an old exam is available for download from the course webpage. This exam does not necessarily resemble this year's exam (also notice that the time allowed and the material covered in this old exam are slightly different from this year's exam).

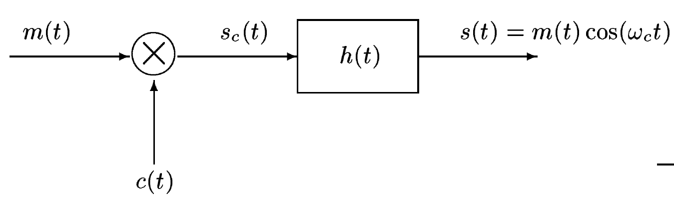
Problem 3.1 (Optional)

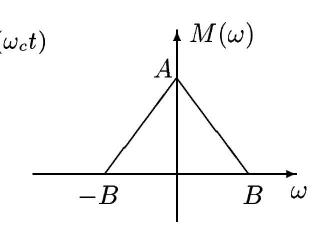
Problem 4.2-1 from Lathi, p. 202.

Problem 3.2

The spectrum of the input signal $m(t)$ and a DSB-SC modulator are shown below. The carrier $c(t)$ available at the multiplier is *distorted* and is given by

$$c(t) = a_1 \cos(\omega_c t) + a_2 \cos^2(\omega_c t) .$$





(a) Determine the spectrum of the signals $s_c(t)$ and $s(t)$.

(b) What constraints should the filter $h(t)$ satisfy so that the transmitted signal $s(t)$ is the desirable one?

(c) What minimum value of ω_c is required for this system to work?

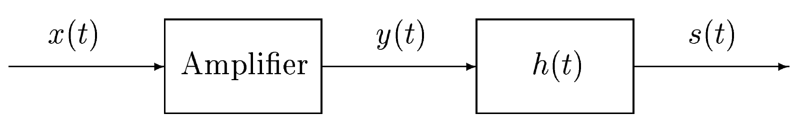
Problem 3.3

Problem 4.2-8 from Lathi, p. 203.

Problem 3.4

A DSB-SC signal $x(t) = m(t)\cos(\omega_c t)$ is amplified before transmission over a channel. Unfortunately, the amplifier is nonlinear with output $y(t)$ related to its input $x(t)$ via the relation

$$y(t) = 100x(t) + x^2(t) \ .$$



(a) Assuming the spectrum of $m(t)$ is limited to $\pm B$ rad/s, find and sketch the spectra of signals $y(t)$ and $s(t)$. (Filter $h(t)$ is the standard bandpass filter used in DSB-SC modulation.)

(b) If we use coherent detection (i.e., assuming we know ω_c exactly), is it possible to recover the signal $m(t)$ at the receiver without distortion? If so, what are the restrictions on the value of ω_c ?

Problem 3.5 (Optional)

A square-law detector is one that uses a nonlinear device to demodulate an amplitude modulated waveform. The output $y(t)$ of this nonlinear device is related to its input $x(t)$ via

$$y(t) = x(t) + \alpha x^2(t) \ ,$$

where α is a constant. If the input to this nonlinear device is an amplitude modulated signal

$$s(t) = A_c[1 + km(t)]\cos(\omega_c t) \ ,$$

find (i) the output $y(t)$ and (ii) the conditions under which $m(t)$ can be recovered exactly from $y(t)$.

Problem 3.6

(a) Problem 4.2-4 from Lathi, p. 202.

(b) Problem 4.3-2 from Lathi, p. 204.

(c) Problem 4.3-4 from Lathi, pp. 204–205.

Problem 3.7

Recall that the complex envelop $\tilde{x}(t)$ of a real signal $x(t)$ is generally complex and can be written in the form

$$\tilde{x}(t) = x_I(t) + jx_Q(t) ,$$

where $x_I(t)$ and $x_Q(t)$ are the in-phase and quadrature components of $x(t)$.

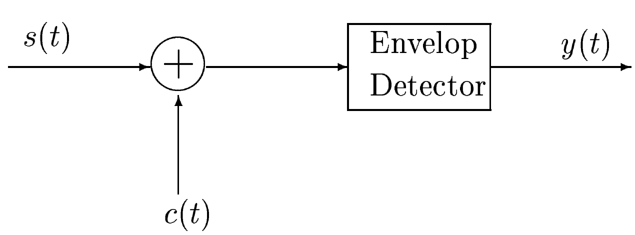
Find the expressions for the in-phase and quadrature components of the following signals:

(a) DSB-SC signal $x(t) = m(t)\cos(\omega_c t + \theta)$.

(b) DSB-TC signal $x(t) = [A + m(t)]\cos(\omega_c t + \theta)$.

Problem 3.8 (Optional)

A DSB-SC signal can be demodulated by an envelop detector if a sufficient amount of carrier is reinserted at the receiver as shown below.



In particular, show that if the carrier $c(t) = A\cos[(\omega_c + \Delta\omega)t + \delta]$ is reinserted in the received DSB-SC signal $s(t) = m(t)\cos\omega_c t$ and the resulting signal is envelop-detected, then the output (after DC blocking) is given by $y(t) = m(t)\cos(\Delta\omega\, t + \delta)$.

Problem 3.9

(a) Problem 4.5-1 from Lathi, pp. 205–206.

(b) Consider the SSB modulated waveform

$$s(t) = m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t \ ,$$

where $m(t)$ is the message signal and $\hat{m}(t)$ is its Hilbert transform. Suppose that $s(t)$ is the input to a square-law device whose output $y(t)$ is then given by

$$y(t) = s^2(t) \ .$$

Show that $y(t)$ contains a frequency component at twice the frequency. Do you think it would be practical to try and recover the message signal $m(t)$ from this component?

Problem 3.10

A VSB-SC signal $s(t) = m(t)\cos\omega_c t - m'(t)\sin\omega_c t$ is coherently demodulated by a locally generated carrier $A\cos[(\omega_c + \Delta\omega)t + \delta]$ as shown below. Find the output signal $y(t)$.