University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 359: Communications I

Fall 2000

Mid-Semester Exam I

Thursday, September 28, 1:30–2:50pm, 161 Everitt Laboratory

READ THESE COMMENTS BEFORE STARTING THE EXAM!!

- This is a **closed-book** exam, but **one** sheet of notes (both sides) is allowed. Calculators should not be necessary, but feel free to use one.
- **Write your name on the answer booklet.**
- There are **four** equally weighted problems for a total of **80 points**.
- A correct answer does not guarantee credit; an incorrect answer does not guarantee loss of credit. **Provide clear explanations, show all relevant work and justify your answers!** If we cannot make sense of your writing or reasoning, you may lose points.
- Read each problem carefully and *think* before performing detailed calculations.
- Only the supplied answer booklet is to be handed in. **No additional pages will be considered in the grading.** You may want to work things through in the blank areas of the exam and then neatly transfer to the answer sheet the work you would like us to look at.

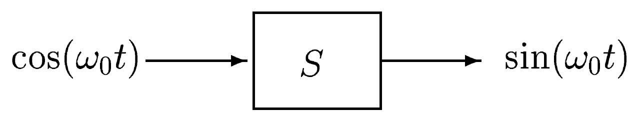
Formulas:

- $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$
- $\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
- $\sin a \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$
- $\cos a \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$
- $\sin a \cos b = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
- $\sin^2\theta + \cos^2\theta = 1$
- $e^{j\theta} = \cos\theta + j\sin\theta$
- $\mathcal{FT}\{e^{-\alpha t}u(t)\} = \frac{1}{\alpha + j\omega}, \quad \alpha > 0$
- $\mathcal{FT}\{\frac{W}{\pi}\text{sinc}(Wt)\} = \text{rect}(\frac{\omega}{2W})$
- $\mathcal{HT}\{\cos(\omega_0 t)\} = \sin(\omega_0 t)$

Problem 1 (20/80, equally weighted parts)

This problem has two independent parts.

(a) For system S shown below, with the indicated input/output pair, determine whether the system could not, could or must be linear time-invariant (LTI). Choose the strongest statement that applies and justify your answer. If you decide that the system could or must be LTI, determine a possible frequency response for it.

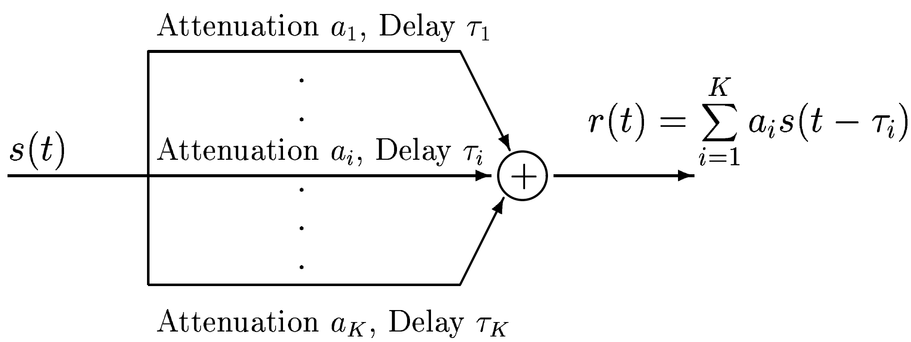


(b) Determine whether the following statement is true or false and justify your answer.

Statement: If $h(t)$ is the impulse response of an LTI system and $h(t)$ is periodic and non-zero, then the system is unstable.

Problem 2 (20/80, equally weighted parts)

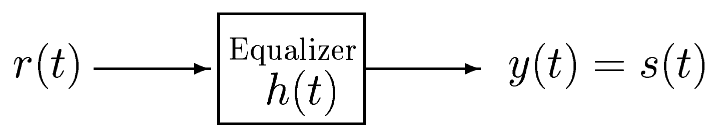
In wireless communications, the signal arriving at the receiver frequently suffers *multipath distortion*. More specifically, the received signal $r(t)$ is the superposition of multiple versions of the transmitted signal $s(t)$, each of which is attenuated and delayed by a different amount.



For the purposes of this problem, assume that the received signal $r(t)$ is given by

$$r(t) = s(t) + \frac{1}{4}s(t - T) ,$$

where $s(t)$ is the transmitted signal and $T > 0$ is a positive delay. The signal $r(t)$ is processed by a linear time-invariant (LTI) system with impulse response $h(t)$ so that its output $y(t)$ is the transmitted signal $s(t)$. This system is called an *equalizer*.



(a) Find the frequency response $H(\omega)$ of the equalizer and determine its phase as a function of ω.

(b) The impulse response of the equalizer can be expressed in the form

$$h(t) = \sum_{k=0}^{+\infty} h_k \delta(t - kT) .$$

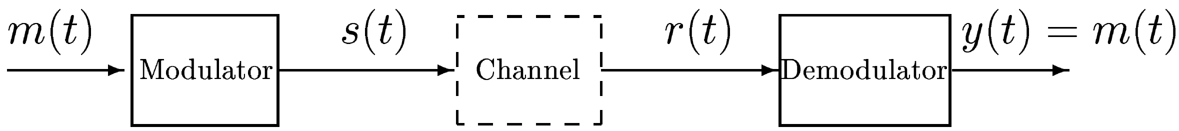
Find h_0, h_1 and h_2.

Hint: Recall that for $|x| < 1$, the following identity is true:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots .$$

Problem 3 (20/80, equally weighted parts)

This problem has four parts that can be solved independently.



The message signal $m(t)$ is transmitted using DSB-SC modulation as shown above. The transmitted signal $s(t)$ is given by $s(t) = m(t)\cos(40000\pi t)$ and the Fourier transform of the signal $m(t)$ is given by

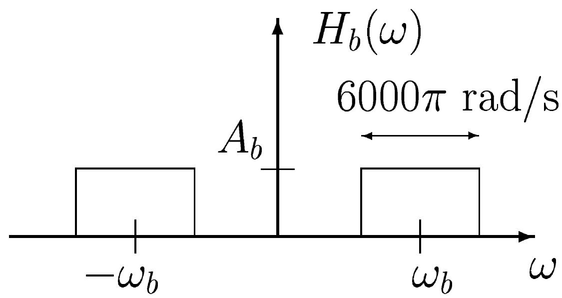
$$M(\omega) = \begin{cases} 1\,, & |\omega| \leq 3000\pi \text{ rad/s} \\ 0\,, & |\omega| > 3000\pi \text{ rad/s}\,. \end{cases}$$

(a) Find and sketch $m(t)$.

(b) Provide an accurate sketch of $S(\omega)$.

In the local hardware store, the only available components are the following:

1. Oscillators that generate the signal $\cos(\omega_0 t)$ at frequency $\omega_0 = 20000\pi$ rad/s.
2. Bandpass filters with frequency response $H_b(\omega)$ as shown below. The gain A_b has to satisfy $0 \leq A_b \leq 10$ and the center frequency ω_b has to satisfy $\omega_b \geq 3000\pi$ rad/s.



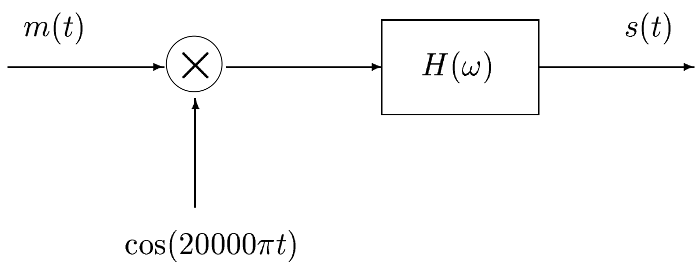
(c) *Using only the components that are available in the local hardware store*, provide an appropriate block diagram for the modulator.

(d) Assuming that the received signal $r(t)$ is undistorted (i.e., $r(t) = s(t) = m(t)\cos(40000\pi t)$) describe whether it is possible to build a coherent demodulator whose output $y(t)$ is equal to $m(t)$, *using only the components that are available in the local hardware store.*

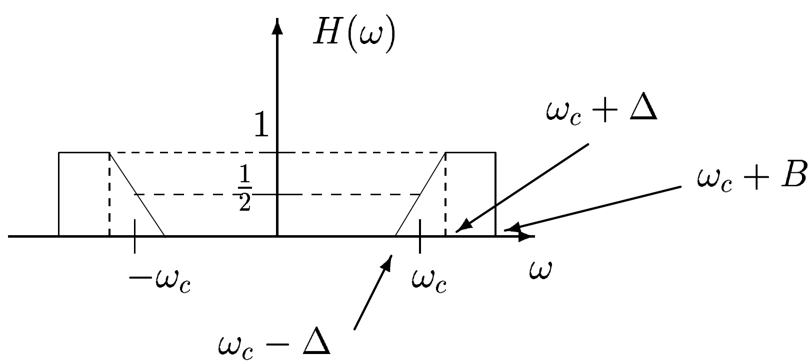
Problem 4 (20/80, equally weighted parts)

This problem has two parts that can be solved independently.

Consider the following VSB modulation scheme.



The frequency response of the VSB filter $H(\omega)$ is shown below, where $\omega_c = 20000\pi$, $W = 500\pi$, and $\Delta = 200\pi$.



Let $m(t) = \cos(300\pi t) + \cos(100\pi t)$.

(a) Find the transmitted VSB-modulated waveform $s(t)$.

(b) Describe a coherent demodulator for $s(t)$ so that its output $y(t)$ equals $m(t)$.