

Problem Set 3

**Group and Phase Delays, Amplitude Modulation, DSB-SC and DSB-TC,
Single-Sideband and Vestigial-Sideband Modulation**

Issued: Thursday, Sept. 15th.

Due: Thursday, Sept. 22nd (beginning of lecture).

Reading from Lathi: Chapter 4, Sections 4.1–4.6.

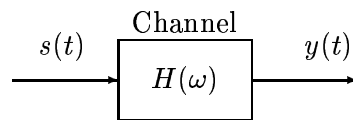
Announcement: The first Mid-Semester Exam will be held on Thursday, October 6th, from 7-9pm in 165 Everitt. The exam will cover all material from the beginning of the term *up to and including* the lecture on Tuesday, September 27th. The corresponding material includes Problem Sets 1 through 4 and Chapters 1, 2, 3, 4 (excluding Sections 4.7–4.9) and 5 (excluding Section 5.5) from Lathi.

During the exam, you can bring an 8.5 × 11-inch double-sided sheet of *handwritten* notes. Calculators are allowed but will not be necessary.

A copy of an old exam is available for download from the course website. This exam does not necessarily resemble this year's exam (also notice that the time allowed and the material covered in this old exam are slightly different from this year's exam).

Problem 3.1

Suppose the modulated signal $s(t) = m(t) \cos(\omega_c t)$ is applied as an input to an LTI communication channel with frequency response $H(\omega)$ as shown below. The modulating signal $m(t)$ is given by $m(t) = \text{sinc}(\frac{\pi t}{T})$ with $(1/T) = 75$ KHz and $\omega_c = 2\pi \times 1300$ Krad/s.



- (a) Make a fully labeled sketch of $S(\omega)$.
- (b) Find a time-domain expression for the output $y(t)$ of the channel if the channel frequency response is

$$H(\omega) = e^{-j\omega(4 \times 10^{-6})} .$$

- (c) Find an *approximate* (but reasonably accurate) time-domain expression for the output $y(t)$ of the channel if the channel characteristics are (not as specified in part (b) but are)

known to satisfy

$$\begin{array}{lll}
 |H(\omega_c/2)| = 1 & |H(\omega_c)| = 1 & |H(2\omega_c)| = 1 \\
 \angle H(\omega_c/2) = -\frac{\pi}{2} & \angle H(\omega_c) = \frac{\pi}{2} & \angle H(2\omega_c) = \frac{3\pi}{4} \\
 \tau_g(\omega_c/2) = 1.2\text{sec} & \tau_g(\omega_c) = 1\text{sec} & \tau_g(2\omega_c) = 0.5\text{sec}
 \end{array}$$

(Note that $\tau_g(\omega)$ denotes the group delay at frequency ω .) State what features of the signal and/or channel make your approximation reasonable.

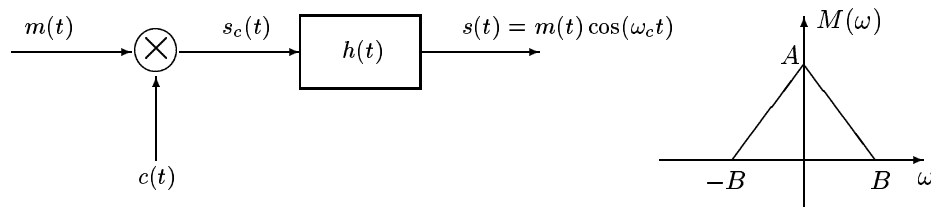
Problem 3.2 (Optional)

Problem 3.5-4 from Lathi, p. 148 (assume that distortionless transmission requires an amplitude variation of less than 4% and a time delay variation of less than 2%).

Problem 3.3

The spectrum of the input signal $m(t)$ and a DSB-SC modulator are shown below. The carrier $c(t)$ available at the multiplier is *distorted* and is given by

$$c(t) = a_1 \cos(\omega_c t) + a_2 \cos^2(\omega_c t) .$$



- Determine the spectrum of the signals $s_c(t)$ and $s(t)$.
- What constraints should the filter $h(t)$ satisfy so that the transmitted signal $s(t)$ is the desirable one?
- What minimum value of ω_c is required for this system to work?

Problem 3.4 (Optional)

Problem 4.2-1 from Lathi, p. 202.

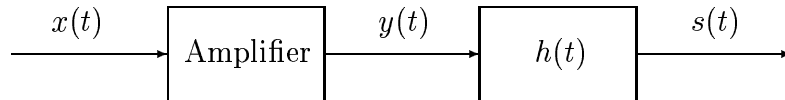
Problem 3.5

Problem 4.2-8 from Lathi, p. 203.

Problem 3.6

A DSB-SC signal $x(t) = m(t) \cos(\omega_c t)$ is amplified before transmission over a channel. Unfortunately, the amplifier is nonlinear with output $y(t)$ related to its input $x(t)$ via the relation

$$y(t) = 100x(t) + x^2(t) .$$



- (a) Assuming the spectrum of $m(t)$ is limited to $\pm B$ rad/s, find and sketch the spectra of signals $y(t)$ and $s(t)$. (Filter $h(t)$ is the standard bandpass filter used in DSB-SC modulation.)
- (b) If we use coherent detection (i.e., assuming we know ω_c exactly), is it possible to recover the signal $m(t)$ at the receiver without distortion? If so, what are the restrictions on the value of ω_c ?

Problem 3.7 (Optional)

A square-law detector is one that uses a nonlinear device to demodulate an amplitude modulated waveform. The output $y(t)$ of this nonlinear device is related to its input $x(t)$ via

$$y(t) = x(t) + \alpha x^2(t) ,$$

where α is a constant. If the input to this nonlinear device is an amplitude modulated signal

$$s(t) = A_c[1 + km(t)] \cos(\omega_c t) ,$$

find (i) the output $y(t)$ and (ii) the conditions under which $m(t)$ can be recovered exactly from $y(t)$.

Problem 3.8

- (a) Problem 4.2-4 from Lathi, p. 202.
- (b) Problem 4.3-2 from Lathi, p. 204.
- (c) Problem 4.3-4 from Lathi, pp. 204–205.

Problem 3.9

Recall that the complex envelop $\tilde{x}(t)$ of a real signal $x(t)$ is generally complex and can be written in the form

$$\tilde{x}(t) = x_I(t) + jx_Q(t) ,$$

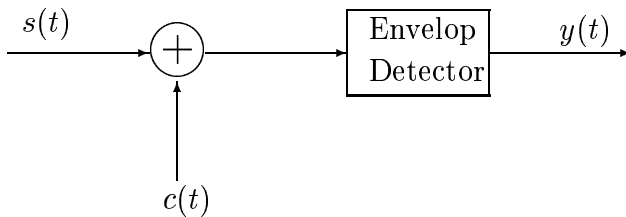
where $x_I(t)$ and $x_Q(t)$ are the in-phase and quadrature-phase components of $x(t)$.

Find the expressions for the in-phase and quadrature-phase components of the following signals:

- (a) DSB-SC signal $x(t) = m(t) \cos(\omega_c t + \theta)$.
- (b) DSB-TC signal $x(t) = [A + m(t)] \cos(\omega_c t + \theta)$.

Problem 3.10 (Optional)

A DSB-SC signal can be demodulated by an envelop detector if a sufficient amount of carrier is reinserted at the receiver as shown below.



In particular, show that if the carrier $c(t) = A \cos[(\omega_c + \Delta\omega)t + \delta]$ is reinserted in the received DSB-SC signal $s(t) = m(t) \cos \omega_c t$ and the resulting signal is envelop-detected, then the output (after DC blocking) is given by $y(t) = m(t) \cos(\Delta\omega t + \delta)$.

Problem 3.11

- (a) Problem 4.5-1 from Lathi, pp. 205–206.
- (b) Consider the SSB modulated waveform

$$s(t) = m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t ,$$

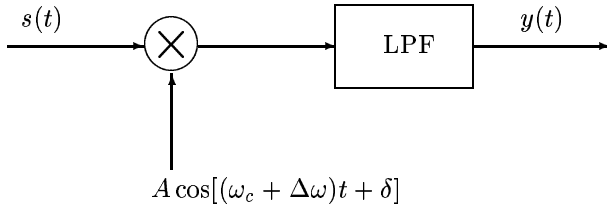
where $m(t)$ is the message signal and $\hat{m}(t)$ is its Hilbert transform. Suppose that $s(t)$ is the input to a square-law device whose output $y(t)$ is then given by

$$y(t) = s^2(t) .$$

Show that $y(t)$ contains a frequency component at twice the frequency. Do you think it would be practical to try and recover the message signal $m(t)$ from this component?

Problem 3.12

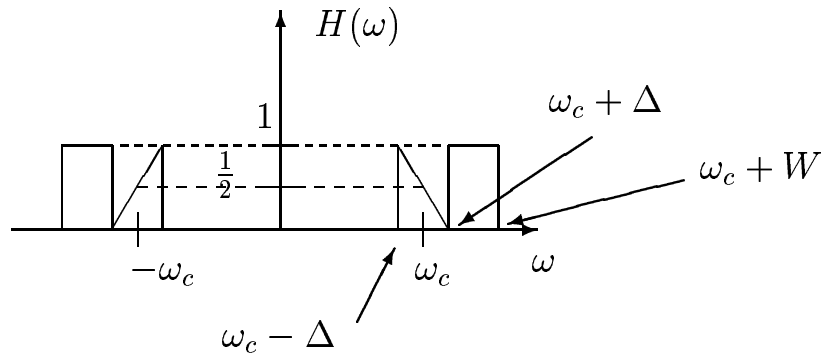
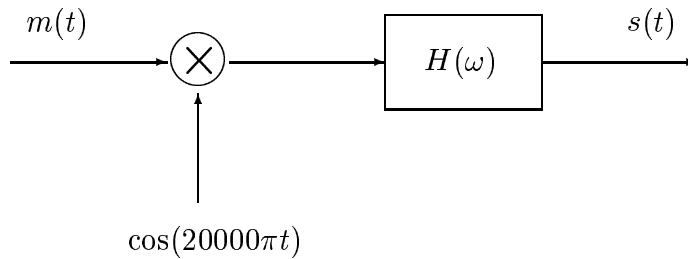
A VSB-SC signal $s(t) = m(t) \cos \omega_c t - m'(t) \sin \omega_c t$ is coherently demodulated by a locally generated carrier $A \cos[(\omega_c + \Delta\omega)t + \delta]$ as shown below. Find the output signal $y(t)$.



Problem 3.13

Consider a VSB-SC modulation scheme with bandpass filter $H(\omega)$ as indicated below where the message signal $m(t)$ is given by $m(t) = \text{sinc}(Wt)$ and the relevant parameters are

$$\omega_c = 20000\pi \text{ rad/s}, \quad W = 1000\pi \text{ rad/s}, \quad \Delta = 200\pi \text{ rad/s}.$$



- (a) Describe a demodulator that receives $s(t)$ as an input and outputs a signal $m_o(t)$ that satisfies $m_o(t) = m(t)$.

(b) The signal $s(t)$ can be expressed as

$$s(t) = \frac{1}{2}m(t) \cos(\omega_c t) - \frac{1}{2}m'(t) \sin(\omega_c t)$$

for an appropriate *real* signal $m'(t)$. Give a frequency-domain expression for $M'(\omega)$ and plot it as a function of ω .