

ECE 434: Random Processes

Spring 2003

Exam 2

Monday, April 14, 2003

Name: _____

- You have 75 minutes for this exam. The exam is closed book and closed note, except that you may consult both sides of two sheets of notes, typed in font size 10 or equivalent handwriting size.
- Calculators, laptop computers, Palm Pilots, two-way e-mail pagers, etc. may not be used.
- Write your answers in the spaces provided.
- **Please show all of your work. Answers without appropriate justification will receive very little credit.** If you need extra space, use the back of the previous page.

Score:

1. _____ (9 pts.)

2. _____ (8 pts.)

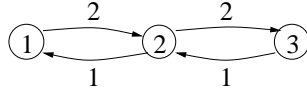
3. _____ (6 pts.)

4. _____ (8 pts.)

5. _____ (9 pts.)

Total: _____(40 pts.)

Problem 1 (9 points) Let X be a stationary continuous-time Markov process with the transition rate diagram shown.



(a) Write down the Q matrix and find the first-order probability distribution π . (So π is a probability vector, representing the distribution of X_t for each t .)

(b) Let X^2 denote the process $X^2 = (X_t^2 : t \in \mathbb{R})$. Is X^2 a Markov process? Justify your answer.

(c) Is $(X_t : t \geq 0)$ a martingale? Justify your answer.

Problem 2 (8 points) Let $(W_t : t \geq 0)$ be a Brownian motion with parameter $\sigma^2 = 1$.

(a) Find $E[W_2 | W_1^3 = 8]$.

(b) Find $E[W_2^3 | W_1 = 1]$.

Problem 3 (6 points) Let $(N_t : t \geq 0)$ be a Poisson process with rate $\lambda > 0$. Find $P[N_2 \geq 3 | N_4 = 4]$.

Problem 4 (8 points) Let $Z = (Z_t : t \in \mathbb{R})$ be a mean zero, stationary Gaussian process with $R_Z(\tau) = e^{-|\tau|}$ and let $V = \int_0^\infty e^{-t} Z_t dt$.
(a) Find $E[V^2]$.

(b) Find $P[V \geq 3]$.

Problem 5 (9 points) Let $B_k : k \geq 0$ be a sequence of independent random variables with $P[B_k = 1] = P[B_k = 0] = P[B_k = -1] = \frac{1}{3}$ for all k . Let $Y = (Y_t : t \geq 0)$ be the continuous time random process such that

- Y has continuous sample paths
- $Y_0 = 0$
- On each interval of the form $[k, k + 1]$, Y has slope B_k

(a) Is Y a Markov process? Justify your answer.

(b) Is Y m.s. continuous? Justify your answer.

(c) Is Y m.s. differentiable? Justify your answer.